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EFFECTS OF GROUND PROXIMITY ON DENSE GAS ENTRAINMENT

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Summary

The effect of the ground on entrainment rates at the top of a dense gas cloud due to atmospheric turbulence is modelled in the laboratory. It is observed that the large eddies generated by grid turbulence are flattened by the bottom of the tank and the mixing that they produce is reduced when the deflection of the interface is comparable with the depth of the dense layer. Experiments relating the deflection of a sharp density interface to the homogeneous turbulence integral length scale are also presented. An emperical relation is derived for the local Richardson number below which ground effects begin to significantly reduce mixing at the top of the cloud.

The cause of this reduction in mixing is examined applying Hunt's theory. For higher local Richardson numbers, the eddies cannot penetrate the dense layer, and entrainment rates are well described by Turner.

1. Introduction

The dispersion of a dense cloud of gas in the atmosphere is controlled by the amount of mixing of the dense gas with the surrounding air. After the dense cloud loses its initial momentum, if any, there follows a buoyancy dominated spreading where the dense cloud slumps, and reduces its height as it behaves like a gravity current. At this time mixing takes place mainly at the advancing front of the cloud but also at the top of the cloud due to turbulence generated within the dense cloud. In a later nearly passive phase, when the lateral extension of the cloud is large the dense cloud is mainly dispersed by atmospheric turbulence, which detrains dense fluid mostly through the top of the cloud. The interface at the top of the cloud is usually sharp and the way to calculate the dilution of the dense cloud at that stage is postulate a top entrainment coefficient.

Entrainment rates obtained in the laboratory are often used to estimate the top mixing calculated by computer models which try to predict the dispersion of a dense gas cloud. Due to the large horizontal extension of the cloud, small

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varations in assumed entrainment rates or mixing coefficients can affect substantially the dilution of the cloud.

There is still some controversy about the empirical relation between the entrainment rate E defined as:

$$E = Ve/u' \tag{1}$$

where Ve is the velocity of penetration of the sharp density interface and u' a turbulent velocity scale, and a local Richardson number, Ri, determined by the density of the cloud and the local turbulence, defined as:

$$Ri = g\,\Delta\rho\,l/\rho\,u'^2$$

where g is the gravity, l the turbulence integral scale, u' the r.m.s. turbulence velocity, $\Delta\rho$ the density difference between the cloud and the air and ρ a mean density.

Turner [1,2], after a series of laboratory experiments with both salt and heat as the stratification producers, proposed: $E \propto Ri^{-3/2}$ for high Peclet number (salt), while a phenomenological theory by Long [3] leads to the relation $E \propto Ri^{-7/4}$. Lately Xuequan and Hopfinger [4] have supported Turner's [1,2] relation. A mechanistic explanation for the $-3/2$ power law was given by Linden [5]. By projecting vortex rings against a sharp interface, he showed that the mixing takes place mostly during the recoil of the interface and that the buoyancy frequency N, see eqn. (5) below, gives the relevant time scale.

In this paper we concentrate on determining, through laboratory experiments, the effect of a plane boundary near the density interface on mixing across the interface. In dense gas dispersion the scale of atmospheric turbulence is often large compared with the depth of the dense cloud it erodes and it is interesting and useful to know under what circumstances entrainment rates at the top of the dense cloud may be affected.

Several authors have considered the effect of a plane bottom boundary on the turbulence near it. Hunt and Graham [6] developed a linear theory for the effect of a rigid boundary introduced into an advected turbulent field. Their results were supported by the wind-tunnel moving floor experiments of Thomas and Hancock [7]. Hunt [8] extended the theory to convective and zero mean flow boundary layers showing that the boundary reduces the r.m.s. vertical velocity, w', and increases the r.m.s. horizontal velocities, u', v', near the boundary. A reduction of w' near a solid boundary had been recorded by McDougall [9] using oscillating grid generated turbulence. The same method of generating turbulence is used in the present experiments. Oscillating grid generated turbulence is a convenient way to produce spatially decaying turbulence. Given a particular grid, it is possible to determine the r.m.s. turbulent velocity, u', and the integral length scale, l, of the turbulence at a particular distance from the plane of oscillation of the grid.

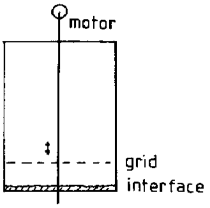


Fig. 1. Diagram of the experimental apparatus used. The grid is oscillated perpendicularly to the interface and driven by a motor.

2. Experimental method

The experiments were carried out in a rectangular perspex tank 46 cm deep and 25.4 cm $\times$ 25.4 cm in cross section, as shown in Fig. 1. Turbulence was generated by oscillating a horizontal grid consisting of 1 cm square bars in a 5×5 array with 5 cm between the centres. The grid was oscillated vertically with a stroke of 1 cm with the frequency ω taking values between 2 Hz and 20 Hz. This tank and grid have been used for mixing experiments by several authors [1,9–11].

2.1 Relation between the displacement of a sharp density interface and the turbulent length scale

It was realized that the turbulence near the wall would be effected by the density interface. As the outer homogeneous turbulence erodes the thin dense layer, the relevant vertical length scale is the displacement of the interface λ.

A set of experiments was done in order to find the effect of the stratification on the defections of the density interface. The same apparatus was used, but the interface was placed in the center of the tank. In these experiments $S = 1$ cm and $\omega = 5$ Hz. The length scale of the turbulence at the interface was varied systematically by placing the grid at different distances from the interface.

By means of slit lighting and fluorescein, photographs were taken of the convolutions of the interface for different local Richardson numbers and the maximum displacement of the interface was averaged for each experiment over the three to five photographs taken at moments of maximum convolution of the interface. The results of these experiments are presented below and are used to interpret the entrainment near the boundary.

2.2 Entrainment far away from a boundary

To compare the entrainment rates obtained for the interface near the bottom of the tank with the values for a similar interface far away from the effects of the boundary, several experiments were made varying the distance between the grid and the interface. These results will be reported elsewhere in detail. It is sufficient to say here that the experiments confirmed Turner's [1,2] results.

The results of these experiments are plotted as filled circles in Fig. 5, together with Turner's results as open circles.

2.3 Entrainment near a boundary

As we are interested in the effects of the bottom on mixing, the grid was placed towards the bottom of the tank. With the tank full of fresh water a thin layer of salty water was carefully injected through an orifice at the bottom of the tank until the layer of dense fluid, usually dyed, had a thickness $d=2$ cm. The distance from the grid to the interface varied throughout the experiments from 5 cm to 13 cm.

Oscillating grid generated turbulence has been studied by several authors [9,10,12]. Thompson and Turner [10] found that at a distance z from the oscillating grid $l \propto z$ and $u' \propto z^{-3/2}$, while McDougall [9] and Hopfinger and Tolly [12] found $l \propto z$ but $u' \propto z^{-1}$. The following expressions were used in our experiment to relate grid parameters to the measured r.m.s. turbulent velocity u' and the integral length scale l, at a vertical distance z from the grid:

$$
\begin{aligned}
u' &= 0.25\, S^{3/2}\, m^{1/2}\, \omega\, z^{-1} \\
l &= 0.1\, z
\end{aligned} \tag{2}
$$

where S is the stroke amplitude of the grid, m the mesh size and ω the grid frequency of oscillation. In all the present experiments $S=1$ cm, $m=$ 5 cm and z was taken as the distance between the center of oscillation of the grid and the center of the thin layer, considered as the mean distance between the grid and the interface.

A total of 51 experiments were made varying ω, z and the density of the thin layer, in order to cover several decades of the value of the local Richardson number defined as:

$$
Ri = g\, \Delta\rho\, l/\rho\, u'^2 \tag{3}
$$

where here $\Delta\rho$ is the density difference between the layers and ρ (1.000 g/cm^3) the reference density. In addition, the distance between the plane of oscillation of the grid and the interface, z, was varied to examine the effect of the integral scale of the turbulence on mixing.

The range of values of l and u' based on unstratified values at the centre of the thin layer were:

$$
\begin{aligned}
&0.1 \text{ cm/s} < u' < 1.5 \text{ cm/s} \\
&0.6 \text{ cm} < l < 1.4 \text{ cm}
\end{aligned} \tag{4}
$$

At the time $t=0$ the grid oscillation was started and visual observations of the erosion of the interface were made both through the sides and bottom of the tank. In most experiments shadowgraph visualization technique was used but in some of the experiments, slit lighting with fluorescein was used to provide a single plane view of the turbulence near the bottom.

The time t taken to erode completely the thin dyed layer of depth d, was determined visually, and the entrainment velocity Ve was calculated as $Ve = d/t$. This is clearly a mean value as the entrainment velocity decreases as the interface moves to a greater distance from the grid. Since the depth, d, of the dense layer is small compared to the distance from the grid, this approximation does not lead to significant errors. The variation in E and Ri due to the change in distance between the grid and the interface z, is shown in form of error bars in the results. The values of u' and l used in the evaluation of E and Ri correspond to a distance from the grid to the middle of the thin layer.

3. Experimental results

3.1 Qualitative results

In most of the experiments viewed by shadowgraph, as soon as the turbulence produced by the grid reached the dense interface, oscillations of the interface were visible. These internal oscillations seemed to define a thin interfacial mixing layer of thickness about 1 cm, but when viewed through the side and by means of slit lighting, it was clear that the integration of the convolutions of the interface across the tank due to the shadowgraph produced this result.

It was observed that in the different experiments, by increasing the total Richardson number, the interface was less easily defected and appreciable mixing events were more intermittent. This is confirmed by the experiments described in Section 2.1.

For lower Richardson numbers, typically $Ri < 10$, it was observed by slit lighting through the side of the tank, that many of the large scale eddies of size comparable or greater than the corresponding integral length scale l were flattened by the bottom surface and extended horizontally. An example of this is shown on Fig. 2b and Fig. 3. For intermediate Richardson numbers $10 < Ri < 100$, this flattening of the eddies due to the bottom only occurred when the layer had been sufficiently eroded because initially the deflection of the interface, λ, is smaller than the depth of the layer, d. As an example, Fig. 2 shows for $Ri = 25$ three stages of the erosion of the thin layer: (a) when mixing is not influenced by the bottom of the tank, (b) when large eddies are flattened, and (c) when the dense fluid near the wall shows viscous sublayer effects.

Viewed through the bottom of the tank with the dense fluid dyed, clear patches could be seen, corresponding to the energetic eddies that deflect the interface sufficiently to collide with the bottom of the tank. It was noticed that the patches were intermittent and their frequency and size decreases as the Richardson number of the experiment increases. No statistical analysis was attempted. Figure 3 shows an artistic impression of the patches viewed through the bottom of the tank. These patches indicate that the bottom of the tank is causing the turbulent eddies that impinge on the interface, to be extended horizontally.

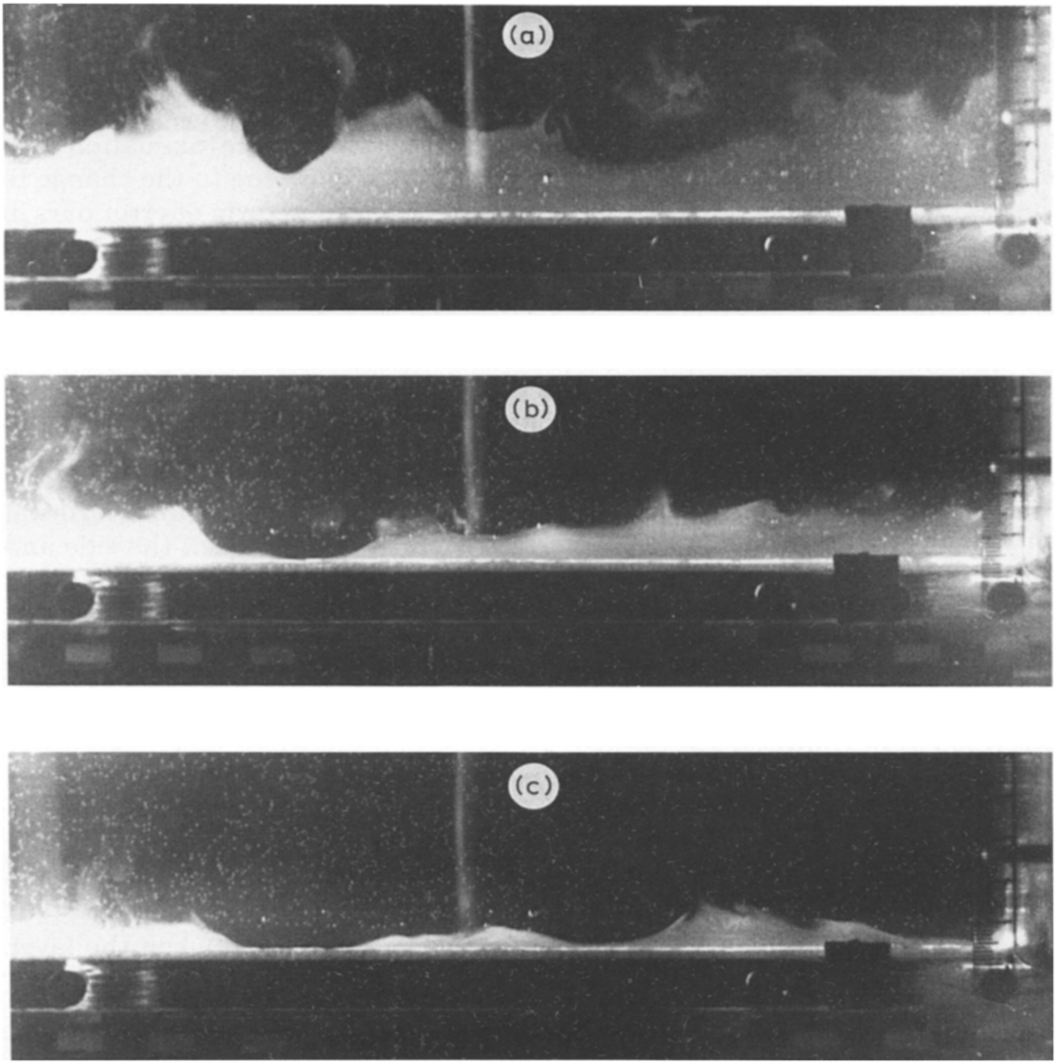


Fig. 2. Thin slit lighting visualization of the erosion of a thin layer. The local Richardson number is 25 and the deflection of the interface by turbulent eddies is significant. (a) Initially the eddies cannot penetrate the dense layer. (b) As the layer is eroded, large eddies are deflected sideways by the bottom of the tank. (c) All eddies are flattened and viscous effects near the wall start being important.

3.2 Quantitative results

The time t, taken to mix completely the thin dense layer with the surrounding turbulent layer was measured. In Fig. 4, the dimensionless time Nt is plotted versus the Richardson number. The buoyancy frequency N was calculated as:

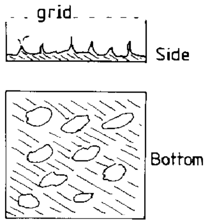


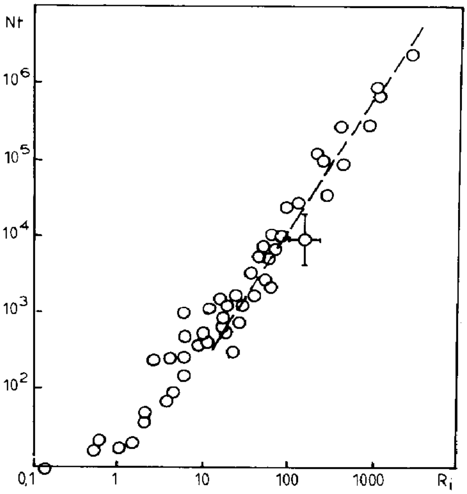
Fig. 3. Observation of the flow structure through the bottom of the tank. Energetic eddies reach the bottom of the tank and eject dyed dense fluid sideways, intermittent patches are seen.

$$N^2 = (- (g/\rho)\ \delta\rho/\delta z) \tag{5}$$

taking as a relevant density gradient that across an interface of thickness $h = 1.5\ l$ [12,13]. We consider the interface as a small linearly stratified region and write:

$$\delta\rho/\delta z = \Delta\rho/h \tag{6}$$

In all the experiments h was between 0.9 cm and 2.1 cm. Figure 4 shows the



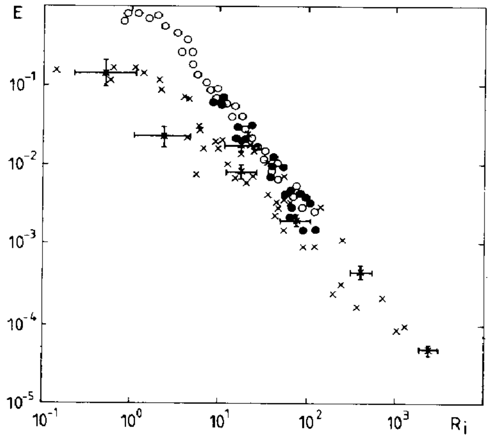


Fig. 4 (left). Total nondimensional time of mixing of the 2 cm deep dense layer. The broken line represents the best fit for points with high Richardson number. Points with lower Richardson numbers do not follow the same power law, indicating a threshold effect of the bottom boundary on mixing.

Fig. 5 (right). Entrainment rates versus local Richardson number for the present experiment of mixing near a boundary ($\times$) compared with Turner's data [1,2] ($\circ$), and experiments on entrainment far away from the boundary ($\bullet$). For low Richardson numbers there is a reduction of entrainment rates due to the effect of the bottom boundary on mixing. The error bars indicate maximum error due to change in turbulence conditions between the top of the dense layer and the bottom boundary.

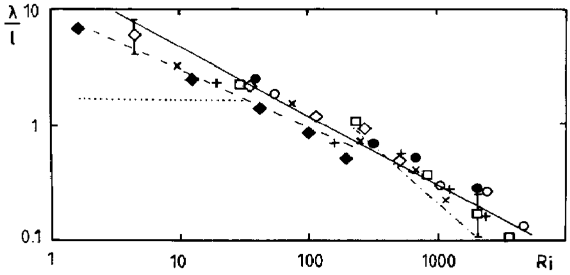


Fig. 6. Maximum displacement of the interface λ non-dimensionalized with the homogeneous turbulence length scale l, versus local Richardson number. The lines indicate the following: -·-·- shows Linden's results [11] converted to local scales using $U = 10\,u'$, $L = l$; - - - - - indicates the power law 1/2, appropriate for low Richardson numbers; ····· indicates the mean maximum displacements for the experiments near a bottom boundary; ——— indicates a 2/3 power law.

best fit for points with $Ri < 35$. Maximum error bars take into account variations of d during each experiment. The equation of best fit is:

$$N\,t = Ri^{1.72}$$

In Fig. 5, the dimensionless entrainment rate E versus local Richardon number is presented and compared with Turner's data [1,2] and the data from the experiment described in Section 2.2 with no nearby boundary. It is observed that points with Richardson numbers greater than approximately 35 are in agreement with Turner's data [1,2] while for lower Richardson numbers entrainment rates are lower for the experiments done near the bottom than those without the influence of the boundary. The error bars in the figure show the variation of the data due to the motion of the interface during the erosion of the dense layer.

4. Deflection of a density interface by turbulence

The results of the experiment described in Section 2.1 on the effect of the Richardson number on the vertical deflection of an interface, are presented in Fig. 6. This figure shows the relation between the maximum displacement of the density interface from its equilibrium position, λ, non-dimensionalized with the unstratified turbulent length scale, and the local Richardson number defined in the same way as in eqn. (3). A relation can be expressed as:

$$\lambda / l = a\,Ri^{-n} \tag{7}$$

where the constants a and n are determined from the experiments as $a = 20 \pm 2$, $n = 0.63 \pm 0.13$, assuming a constant exponent n. The data suggest that the exponent n varies slightly with the Richardson number increasing for higher Richardson numbers. A tentative relation is $n = 1/2$ for $Ri < 10$, $n = 2/3$ for $10 < Ri < 100$ and $n = 1$ for $Ri > 100$.

It is possible to relate λ to the turbulent vertical velocity w' at the interface if we assume that there is a linear density profile in the immediate neighborhood of the nominally sharp interface. Then:

$$\lambda \propto w'/N \tag{8}$$

where N is the buoyancy frequency defined in the same way as in eqn. (5).

Using the experimental results expressed in eqn. (7) and considering an alternative definition for the local Richardson number: $Ri = (N\,l/u')^2$, we can express:

$$w' \propto a\ (N\,l)^{1-2n}\ u'^{2n} \tag{9}$$

and relate the vertical velocity at the interface to the unstratified velocity as:

$$w'^2/u'^2 \propto a^2\ Ri^{1-2n} \tag{10}$$

From this equation we observe that if $n=1/2$ as seems to be the case for lower Richardson numbers, there is no effect on the turbulence near the interface. If $n=1$ as found by Linden [5], the dependence is:

$$w'^2 \propto u'^2\ Ri^{-1} \tag{11}$$

Considering the global value found in the present experiment, valid also for intermediate Richardson numbers, i.e. $n=2/3$, gives:

$$w'^2 \propto u'^2\ Ri^{-1/3} \tag{12}$$

which agrees with the theory given by Carruthers and Hunt [14] for velocity fluctuation near an interface between a turbulent region and a stably stratified layer.

5. Discussion

The data presented above indicates a reduction of entrainment rates near a bottom boundary for low Richardson numbers. This is interpreted as being the effect of the bottom boundary on the mixing process. When the relevant penetration depth, λ, related to the integral length scale of the homogeneous turbulence is comparable with the depth of the layer, for low Richardson numbers the deflection of the interface is stopped by the boundary, hampering the recoil and ejection of dense fluid which contributes to mixing.

When the bottom boundary is near the interface and the mean turbulent displacement of the interface is greater than the depth of the dense layer, $d<\lambda$, in addition to the reduction in vertical velocity produced by the density interface, there is an effect of the wall on the turbulence. Thomas and Hancock [7] found experimentally that in a homogeneous fluid

$$w'^2 \propto u'^2\ (d/l)^{2/3} \tag{13}$$

where d denotes here the distance from the boundary.

The explanation for this relation given there was in terms of eddies impacting on the wall. The experiments done by Thomas and Hancock [7], as well as the analysis of Hunt and Graham [6] strictly applies to a homogeneous turbulent flow which passes over a semi-infinite rigid surface fixed in space but moving at the same velocity as the flow.

Hunt [8] extended the theory of turbulence near a wall to include the effect of convective or zero mean flow turbulence. He shows that near the wall the horizontal velocity fluctuation u', v' grows at the expense of the vertical component w'. Defining one-dimensional spectra for each component of turbulent velocity, Hunt [8] calculated that the vertical velocity spectrum decreases near the boundary for low wave numbers, $k < 1/L$, L being the integral length scale of the turbulence, but remains the same for high wave numbers $k < 1/d$, where d is here the distance from the boundary.

If $d < \delta$, where δ is the depth of the viscous sublayer, there are no significant viscous losses to the wall and the horizontal velocity spectra increases for low wavenumbers. This indicates that the boundary effects mainly the large eddies of size $L < d$.

Figure 7 shows a compilation by Hunt [8] of theory and experiments of the turbulence near a wall. They confirm the transfer from vertical to horizontal motions produced by the wall. Horizontal turbulence velocity components are less efficient than vertical fluctuations in mixing dense fluid at an interface which would explain the reduction in entrainment near the wall.

Figure 8 shows three different mixing processes, depending on the depth of the dense layer and its relation to the integral length scale of the turbulence. In Fig. 8a, the depth of the layer d is greater than the deflection of the interface λ, produced by turbulence of eddy length scale l, so the mixing is not affected by the presence of the boundary. In Fig. 8b both scales d and λ are comparable so the eddies are flattened against the boundary and lose some of their upward velocity as they are squashed, thus decreasing the mixing rate. In Fig. 8c it is shown that when the depth of the layer is comparable with the viscous sublayer depth, viscosity is the relevant parameter and there is no influence of the Richardson number on mixing rates.

An estimate of the depth of the viscous sublayer δ can be obtained using the relation for the growth of the viscous boundary layer in oscillating flow [17]:

$$\delta \propto (\nu t)^{1/2} \tag{14}$$

Considering that the flow oscillates with period $t = l/u'$, given by the turbulence near the wall, it is possible to calculate δ for each experiment. For all the experiments, the depth of the viscous sublayer was bounded: 0.06 cm $< \delta <$ 0.27 cm.

The reduction in entrainment rates only takes place at lower Richardson numbers because the relevant eddy scale is the deflection of the interface which

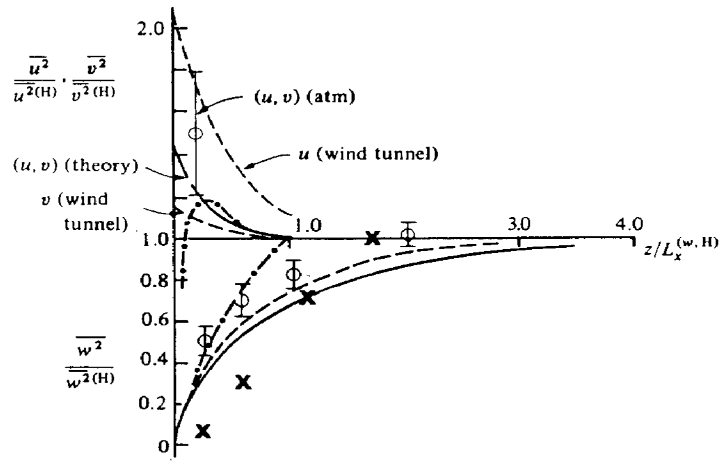


Fig. 7. Compilation by Hunt [8] of the variation of the turbulent velocities near a solid boundary for convective and shear-free boundary layers: Φ, convective boundary layer [15]; $\times$, mixing-box turbulence near a rigid lid [9]; – – – –– wind tunnel moving floor [7]; ———, theory [6]; ·–·–· laboratory convection [16]. Superscript (H) refers to the values in the homogeneous turbulence outside the boundary layers.

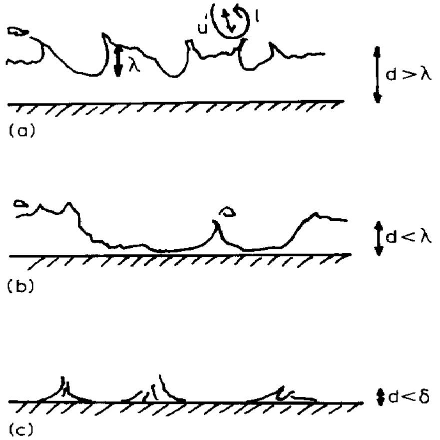


Fig. 8. Different situations in thin dense layer mixing. (a) When the eddy size l and the maximum deflection of the interface λ are smaller than the depth of the layer d, there is no influence of the ground on mixing. (b) When both scales λ and d are comparable, eddies are "squashed" against the ground and lose mixing efficiency. (c) For dense layers of depth comparable with the viscous sublayer δ mixing is reduced, but this effect in not important in atmospheric dense gas dispersion.

decreases with Richardson number. The turbulence at higher Richardson numbers hardly deflects the interface so there is no reduction in mixing until the thickness of the layer is comparable with the viscous sublayer.

6. Conclusions

It is seen that a solid boundary near a density interface can affect the turbulent mixing taking place through the interface. Several authors have shown the reduction in vertical turbulent velocity near a wall, and furthermore that the larger scales of turbulence are the most affected by the reduction. Since the larger vertical scales are more effective at mixing in a stratified fluid, the present observations indicate that the blocking of these scales by the bottom boundary will reduce mixing rates for low Richardson numbers.

With the atmospheric dispersion of a dense gas cloud in mind, it is possible to give a simple relation to determine whether the proximity of the ground will reduce the mixing at the top of the cloud. If the vertical thickness of the cloud, d, is less than or comparable with λ, the presence of the ground will reduce mixing at the top of the cloud. The expression:

$$Ri \leqslant (a\, l/d)^{1/n} \tag{15}$$

gives the condition for the ground to affect mixing at the top of the dense gas cloud.

In the experiments on entrainment near a boundary presented above, where the integral length scale of the turbulence is comparable with the depth of the dense layer, we found that reduction in entrainment takes place for $Ri \leqslant 35$. From the laboratory experiments the values of the constants that could be used in inequality (15) are: $a = 20 \pm 2$ and $n = 0.63 \pm 0.13$.

It is possible to give an expression for the reduced entrainment rate if we consider $l \simeq d$, but in a general case the reduction in E will depend on the parameters Ri and l/d. Future experiments which include the variation of l/d are planned. A comparison of field data with the experimental results presented here would determine further the conditions under which a reduced entrainment rate applies.

Acknowledgments

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